Cambridge Mathematics School Admissions Test Mark Scheme

Specimen Paper 2

Mark Scheme

Marking instructions

- Each question in sections A and B scores 2 marks for the correct answer or zero for no answer, the wrong answer or more than one answer.
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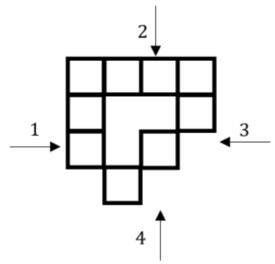
- For section C
    - M marks are for working and are given for a correct method, clearly shown even if there are some errors of arithmetic.
    - A marks are for the correct answer from correct working and can only be given if all the M marks so far in that part of the question have been earned.
    - B marks are independent marks.
    - Candidates may use any correct method; if this method is not in the mark scheme, award marks in a way that is as similar as possible to the methods shown in the mark scheme.

Section A

Number	Solution	Mark	Guidance
			Either 2 or zero for each question on Section A.
1	**C** A circle of radius 3 cm	**2**	**Example reasoning** Area of A is 24, B is 25, C is 9π which is a little over 27, D is 27 and E is 8π
2	**D** $3\frac{1}{3}$ square units	**2**	**Example reasoning** When $x = 0$, $y = -2$. When $y = 0$, $x = {}^{10}/_{3}$ Area $= \frac{1}{2} \times 2 \times {}^{10}/_{3} = {}^{10}/_{3} = 3\frac{1}{3}$
3	**A** 2×10^7	**2**	**Example reasoning** This can be rewritten as $\frac{32\times10^5\times45\times10^3}{72\times10^2}$. This simplifies to 20×10^6. $= 2 \times 10^7$ in SI form
4	**D** $9:4$	**2**	**Example reasoning** PQR is similar to SQT with length scale factor $\frac{2}{3}$. Let the height of PQR perpendicular PR to be h. The height of SQT will be $\frac{2}{3}h$ and $\mathrm{ST} = \frac{2}{3}\mathrm{PR}$. Area of $\mathrm{PQR} = \frac{1}{2}\mathrm{PR}h$, area of $\mathrm{SQT} = \frac{1}{2}.\frac{2}{3}\mathrm{PR}\,.\frac{2}{3}h = \frac{1}{2}.\frac{4}{9}\mathrm{PR}h$ ratio $1:\frac{4}{9}$ i.e. $9:4$
5	**C** $£20$	**2**	**Example reasoning** The youngest receives $\frac{4}{5} \times \frac{5}{6} = \frac{2}{3}$ of the eldest's amount $\frac{2}{3}e + \frac{5}{6}e + e = 50$ multiply by 6 $4e + 5e + 6e = 300$ giving $e = 20$
6	**D** $5 \times \frac{p^3}{10q^2} \times \frac{3}{(pq)^3}$	**2**	**Example reasoning** Considering indices for q: A is $q^{4-3} = q$, B is $q^{3-2} = q$, C is q, D is $q^{-2-3} = q^{-5}$, E is $q^{-3+5-1} = q$

Number	Solution	Mark	Guidance
7	**A** Their sum must be a multiple of 3	**2**	**Example reasoning** If the first number is n then the 3 consecutive numbers add up to $3n+3$ This is a multiple of 3 for all n. Counter examples can be found for statements **B**, **C**, **D** e.g. **B** any even n, **C** any odd n, **D** $n=2$ **E** cannot be true if **A** is true.
8	**A** $\frac{1}{2}$	**2**	**Example reasoning** $\mathrm{P}(3)+\mathrm{P}(6)=\frac{1}{3}$, , $\mathrm{P}(2)=\frac{1}{12}$, $\mathrm{P}(5)=\frac{1}{12}$ $\mathrm{P}(2)+\mathrm{P}(3)+\mathrm{P}(5)+\mathrm{P}(6)=\frac{1}{3}+\frac{1}{12}+\frac{1}{12}=\frac{1}{2}$ So $\mathrm{P}(1)+\mathrm{P}(4)=1-\frac{1}{2}=\frac{1}{2}$
9	**B** 4	**2**	**Example reasoning** The only points satisfying $x>0$, $y>0$ and $x+y<5$ are $(1,1), (1,2), (1,3), (2,1), (2,2)$ and $(3,1)$ From these, all but $(1,2)$ and $(1,3)$ satisfy $y<2x$ so there are 4 points
10	**B** $3\left(1-\frac{\pi}{4}\right)$	**2**	**Example reasoning** A quarter of the circle has area $\frac{\pi}{4}$. The small square, with side OQ, minus a quarter circle has area $1-\frac{\pi}{4}$. There are two complete sections like this and two sections that are half of this making 3 sections in total so the total shaded area is $3\left(1-\frac{\pi}{4}\right)$

Section B

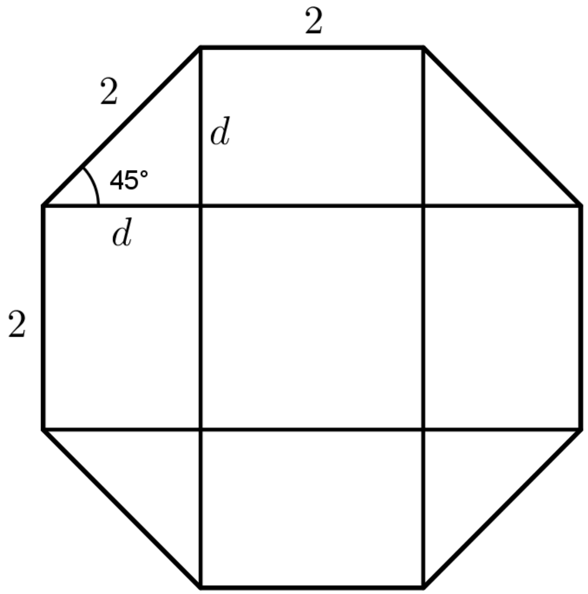
Number	Solution	Mark	Guidance
			Either 2 or zero for each question on Section B.
11	**D** 20	**2**	**Example reasoning**  If the elevation is in direction 1 the maximum number of cubes is $4 \times 3 + 2 \times 2 + 2 \times 1 + 1 \times 2 = 20$ From direction 2 it is $2 \times 3 + 2 \times 2 + 2 \times 1 + 3 \times 2 = 18$ From direction 3 it is $1 \times 3 + 2 \times 2 + 2 \times 1 + 4 \times 2 = 17$ From direction 4 it is $3 \times 3 + 2 \times 2 + 2 \times 1 + 2 \times 2 = 19$
12	**C** 6	**2**	**Example reasoning** For the numbers to still be prime when the digits are swapped, the tens digit must be 1, 3, 7 or 9. In the given range, the only primes that do this are 31 and 37. $37 - 31 = 6$

Number	Solution	Mark	Guidance
13	**E** The country with the largest population won the most medals.	**2**	**Example reasoning** **A** is not true as the smallest population (< 1 million) won 1 medal but the three larger countries from the table won 0 medals. **B** is not true as there are points showing smaller countries having more medals than some larger countries. **C** is not true as one country with just under 10 million population won 17 medals **D** cannot be a conclusion as relative wealth is not indicated **E** is true, the largest country with a population of 44 million won 20 medals (the most)
14	**E** 176	**2**	**Example reasoning** There are 11 numbers starting with 1 from 1 to 99 (1 and $10-19$). Every number from $100-199$ starts with 1 so there will be $11+n-99=n-88$ numbers starting with 1 from 1 to n (for the given values of n). $\frac{n-88}{n}=\frac{1}{2}$ gives $n=176$
15	**C** Around the same amount of rain fell in Beijing in May, June and July of 2015 as fell in August September and October of 2015.	**2**	**Example reasoning** 1987 MJJ $\approx 2.1+3+3.5=8.6$ ASO$\approx 8+1.1+0.5=9.6$ Total $8.6+9.6=18.2$ 2015 MJJ $\approx 1.1+1.6+3.7=6.4$ ASO$\approx 2.8+3+0.6=6.4$ Total $8.6+9.6=12.8$ A no, $8.6<9.6$ B no, $18.2\neq 2\times 12.8$ C yes, $6.4=6.4$ (or close values) D no, $9.6\neq 6.4$ E no, not wettest in 2015

Number	Solution	Mark	Guidance
16	**B** 7	**2**	**Example reasoning** There is 1 way that the total of the first two digits is 1 (10) and 2 ways for the total of the last two digits to be 1 $(10$ and $01)$ giving 1010 and 1001 For a total of 2: 1 way for the first two (11) and 3 for the last two $(20$, 11 and $02)$ giving 1120, 1111, 1102. For a total of 3: 1 way for the first two (12) and 2 for the last two $(12$ and $21)$ giving 1212, 1221. Giving $2+3+2=7$
17	**A** $2^2 \times 3 \times 5^2$	**2**	**Example reasoning** 6000 does not divide by 9 so eliminate B and D 4500 does not divide by 8 so eliminate C The two remaining options have $2^2 \times 3$ as factors Dividing all three by $(2^2 \times 3) = 12$ gives 450, 375, 500 All are divisible by 25 but 450 is not divisible by 125 so eliminate E. Alternatively $\text{hcf}(5\ 400, 4\ 500, 6\ 000) = 2^2 \times 3 \times 5^2$
18	**E** 120	**2**	**Example reasoning** The sequence is $(n+1)^2 - 1$ (can cut up and reassemble to $n+1$ sided squares with a 1×1 square missing) $(10+1)^2 - 1 = 121 - 1 = 120$
19	**E** 100	**2**	**Example reasoning** Let a be the number of starfish, b the number of shrimp and c the number of octopuses $a+b+c=200$ $4a+6b+8c=1400$ $4a+6b+8c=1400$ $\underline{6a+6b+6c=1200}$ $-2a \qquad +2c = 200$ $c-a=100$

Number	Solution	Mark	Guidance
20	**B** 25%	**2**	**Example reasoning** $\frac{1}{a}-\frac{1}{b}=\frac{3}{5}\left(\frac{1}{a}+\frac{1}{b}\right)$ $\frac{5}{a}-\frac{5}{b}=\frac{3}{a}+\frac{3}{b}$ $\frac{2}{a}=\frac{8}{b}$ $2b=8a$ $a=\frac{1}{4}b$

Section C

Number	Solution	Mark	Guidance
21			
	$d = 2\sin 45° = \sqrt{2}$ (alternatively $d = 2\cos 45°$ or $2d^2 = 4$)	A1	Award if d is stated as $\sqrt{2}$ or the correct length is marked as $\sqrt{2}$ on a diagram. Allow $\frac{2}{\sqrt{2}}$ for $\sqrt{2}$
	Area of octagon $= 4 \times \frac{1}{2} \times \sqrt{2} \times \sqrt{2} + 4 \times 2 \times \sqrt{2} + 2 \times 2$ $= 8 + 8\sqrt{2}$	M1 A1	Award M1 for area of octagon using their values. A1 for $8 + 8\sqrt{2}$ or equivalent
	Area of square $= 2 \times 2 = 4$ Shaded area $= 8 + 8\sqrt{2} - 4 = 4 + 8\sqrt{2}\ \text{cm}^2$	A1 B1	Allow equivalent using $\frac{2}{\sqrt{2}}$ for $\sqrt{2}$ i.e. $4 + \frac{16}{\sqrt{2}}$ Allow ft from incorrect area of octagon only. B1 for correct units.

<table>
<tr><th>Number</th><th>Solution</th><th>Mark</th><th>Guidance</th></tr>
<tr><td></td><td>Alternative method
$d = 2\sin 45° = \sqrt{2}$
(alternatively $d = 2\cos 45°$ or $2d^2 = 4$)

Shaded area $= 4 \times \left(2 \times \sqrt{2}\right) + 4 \times \left(\frac{1}{2} \times \sqrt{2} \times \sqrt{2}\right)$

$= 4 + 8\sqrt{2}\ \text{cm}^2$</td><td>A1

M1 A1

A1 B1</td><td>Award if d is stated as $\sqrt{2}$ or the correct length is marked as $\sqrt{2}$ on a diagram. Allow $\frac{2}{\sqrt{2}}$ for $\sqrt{2}$.

M1 evidence of splitting shaded shape into rectangles and triangles. A1 if these are correct.

Allow equivalent using $\frac{2}{\sqrt{2}}$ for $\sqrt{2}$ i.e. $4 + \frac{16}{\sqrt{2}}$

Allow ft from incorrect area. B1 for correct units.</td></tr>
<tr><td></td><td></td><td>[5]</td><td></td></tr>
<tr><td>22 a)</td><td>If the red dice is a 2 then the other 2 must sum to $4, 7$ or 10
<table>
<tr><th colspan="2">$B + Y = 4$</th><th colspan="2">$B + Y = 7$</th><th colspan="2">$B + Y = 10$</th></tr>
<tr><td>B</td><td>Y</td><td>B</td><td>Y</td><td>B</td><td>Y</td></tr>
<tr><td>1</td><td>3</td><td>1</td><td>6</td><td>4</td><td>6</td></tr>
<tr><td>2</td><td>2</td><td>2</td><td>5</td><td>5</td><td>5</td></tr>
<tr><td>3</td><td>1</td><td>3</td><td>4</td><td>6</td><td>4</td></tr>
<tr><td></td><td></td><td>4</td><td>3</td><td></td><td></td></tr>
<tr><td></td><td></td><td>5</td><td>2</td><td></td><td></td></tr>
<tr><td></td><td></td><td>6</td><td>1</td><td></td><td></td></tr>
</table></td><td>B3</td><td>Award 3 marks for all 12 correct possibilities listed with no incorrect possibilities

Award 2 marks for 9 - 11 correct possibilities

Award 1 mark for 5 - 8 correct possibilities</td></tr>
</table>

<table>
<tr><th>Number</th><th>Solution</th><th>Mark</th><th>Guidance</th></tr>
<tr><td>22 b)</td><td>For $B + Y = 4$, $R + B + Y = 6$
For $B + Y = 7$, $R + B + Y = 9$
For $B + Y = 10$, $R + B + Y = 12$
The only ones that give a square number are $B + Y = 7$
<table>
<tr><th colspan="2">$B + Y = 7$</th></tr>
<tr><td>B</td><td>Y</td></tr>
<tr><td>1</td><td>6</td></tr>
<tr><td>2</td><td>5</td></tr>
<tr><td>3</td><td>4</td></tr>
<tr><td>4</td><td>3</td></tr>
<tr><td>5</td><td>2</td></tr>
<tr><td>6</td><td>1</td></tr>
</table></td><td>B2</td><td>Award 2 marks for all 6 correct possibilities listed with no incorrect possibilities
Award 1 mark for 3 - 5 correct possibilities or ft selecting correct possibilities from their answer to (a)</td></tr>
<tr><td></td><td></td><td>[5]</td><td></td></tr>
</table>

Number	Solution	Mark	Guidance
23 a)	$1 + 2 + 3 + \cdots + 12 = 78$	**A1**	
23 b)	The sum of the 4 edges will use the numbers $10, 11, 12$ and n twice and everything else once $(1 + 2 + 3 + \cdots + 12) + 10 + 11 + 12 + n$ $= 78 + 33 + n$ $= 111 + n$	**M1 A1**	M1 for using the corner values twice A1 for correct simplification to $111 + n$
23 c)	The total along each edge is the same and must be an integer. There are 4 edges hence the overall sum is a multiple of 4.	**B1**	Any correct explanation that identifies that the total for each side can be found by dividing by 4.
23 d)	$\frac{111 + n}{4} = 27 + \frac{n + 3}{4}$ $n + 3$ is a multiple of 4 $n + 3 = 4$ gives $n = 1$ $n + 3 = 8$ gives $n = 5$ $n + 3 = 12$ gives $n = 9$ The next value would be greater than 12.	**M1 A2**	M1 for any correct strategy (using the equation shown or by trying values for n) A2 for identifying all 3 correct values for n A1 for identifying 2 of the correct values for n

23 e)	For $n = 1$ the total along each edge is $27 + \frac{4}{4} = 28$ The available numbers to complete the square once the corners have been inserted are $2, 3, 4, 5, 6, 7, 8$ and 9. The left hand column starts with a 12 and finishes with 11 so the other two numbers must add up to 5. This can only be done using the 2 and 3. The top row starts with a 12 and ends with a 10 so the other two numbers add up to 6. None of the remaining numbers will do this so $n = 1$ does not work.	**B1**	Any correct explanation identifying that the 4 totals can not be made using the values available
	For $n = 5$ the total along each edge is $27 + \frac{8}{4} = 29$ The available numbers to complete the square once the corners have been inserted are $1, 2, 3, 4, 6, 7, 8$ and 9. The left hand column needs 6 more to make this total. This can only be done using the 2 and the 4. The top row needs 7 more to make 29. This must use the 6 and the 1. The right hand column needs 14 more to make 28. No pair of numbers from the remaining digits ($3, 7, 8$ and 9) add up to 14 so $n = 5$ does not work.	**B1**	Any correct explanation identifying that the 4 totals can not be made using the values available
	For $n = 9$ the total along each edge is $27 + \frac{12}{4} = 30$ The available numbers are $1, 2, 3, 4, 5, 6, 7$ and 8. The left hand column needs 7 more to make 30. This can be done using the 6 and the 1. The top row needs 8 more to make 30. This can be done using the 5 and the 3. The right hand column needs 11 more to make 30. This can be done with the 7 and the 4. The bottom row needs 10 more to make 30. This can be done using the 8 and the 2. $n = 9$ is the only one that works.	**B1**	Confirming correctly that $n = 9$ does give a possible solution.
		[10]	